ON CERTAIN EXACT SOLUTIONS OF UNSTEADY TWO-DIMENSIONAL GASDYNAMICS

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References [1,2] investigated the unsteady two-dimensional flows of a polytropic gas with rectilinear characteristics in the x_1, x_2, t space constant values of the velocity components u_1, u_2 and of the velocity of sound along straight lines.

In [3] the determination of solutions of this type of flows when shock waves were present was investigated under the assumption that the motion behind the front of the wave is isentropic. The basic property of shock waves in the types of flow indicated is the constancy of their intensity for both isothermal and adiabatic gases. The form of the front of the shock wave may, generally speaking, be arbitrary. (The background on which a shock wave is propagating is assumed to be a stagnant polytropic gas with nonvanishing density and pressure.)

The remarks presented in this paper are the continuation of that work. Here the methods, derived in [3], are applied to the establishment of certain specific solutions of the equations of two-dimensional gasdynamics. It should be noted that in [3] there is an error. In the example under investigation in [3], for shock waves acquiring at a given instant the form of an ellipse, it was stated erroneously that the local solution is valid in the region behind the wave, as the latter is propagated along the uniform background. In reality the solution derived for the hyperbolic case defines some background disturbances in front of the shock wave, as it moves in such a manner that a stagnant region remains behind the wave.

Section 1 of this paper treats the problems of finding solutions in the region between the curvilinear shock wave and its supporting curvilinear piston, along which the pressure is constant in time. A generalization of the well known "automodel" solution of Sedov is obtained for the problem of the expansion of a cylindrical piston at constant velocity into a gas [4].

Section 2 investigates flows behind propagating curvilinear normal detonating waves; the solutions obtained are valid in a region behind the detonating wave which is bounded on one hand by the front of the detonating wave, and on the other either by a weak discontinuity (similar to the case found in the one-dimensional automodel solution of Zel'dovich [5]), or else by the limit line, which is the line of degeneration of the velocity hodograph. The stability and the uniqueness of these solutions are not considered in this paper.

The system of equations, which describes the types of flow under consideration, has the form (this system of equations is somewhat different from the one obtained in [3]):

$$\Delta_1 = \left(r + \frac{\gamma - 1}{2}\,\theta\theta'\right)\cos\varphi, \qquad \Delta_2 = \left(r + \frac{\gamma-1}{2}\,\theta\theta'\right)\sin\varphi \tag{0.1}$$

$$r^2\,\frac{\partial^2\Phi^\circ}{\partial r^2} + (1-\theta'^2)\left(\frac{\partial^2\Phi^\circ}{\partial\varphi^2} + r\,\frac{\partial\Phi^\circ}{\partial r}\right) = 0 \tag{0.2}$$

$$(\gamma-1)\,\theta\,(\theta' + r\theta'' - \theta'^3) + r\,(\gamma-3)\,\theta'^2 + 4r = 0 \tag{0.3}$$

$$x_i - \Delta_i t = \alpha_i \qquad (i = 1,2) \tag{0.4}$$

where Φ is the velocity potential and γ is the adiabatic index.

$$\theta = \frac{2}{\gamma-1}\,c, \qquad u_1 = r\cos\varphi, \qquad u_2 = r\sin\varphi$$

$$\Phi^\circ = \alpha_1 u_1 + \alpha_2 u_2 - \Phi, \qquad \frac{\partial\Phi^\circ}{\partial u_1} = \alpha_1, \qquad \frac{\partial\Phi^\circ}{\partial u_2} = \alpha_2$$

For the total differential equation, (0.3) one specifies Cauchy problem where θ_0' and θ_0 are determined on the front of the shock wave from the Hugoniot conditions and the condition

$$D = \pm\,\frac{u_1\Delta_1 + u_2\Delta_2}{\sqrt{u_1^2 + u_2^2}} = \text{const} \tag{0.5}$$

where D is the normal velocity of the front of a shock wave.

The front of the shock wave is given by the equation

$$\alpha_2 = f(\alpha_1) \tag{0.6}$$

where α_1 and α_2 are parameters which characterize the configuration of the linear characteristics in the $x_1 x_2 t$ space. The initial conditions for Equation (0.2) are as follows

$$\frac{\partial\Phi^\circ}{\partial r} = \cos\varphi f'^{-1}(-\cot\varphi) + \sin\varphi f\,[f'^{-1}(-\cot\varphi)] = l(\varphi) \quad \text{for } r = u_{1n} \tag{0.7}$$

$$r\,\frac{\partial\Phi^\circ}{\partial r} - \Phi^\circ = 0 \quad \text{for } r = u_{1n} \tag{0.8}$$

where u_{1n} is the velocity of the gas in front of the wave, f'^{-1} is the inverse function of f'. After separation of variables in (0.2), assuming $\Phi^{\circ} = \psi(\varphi)\chi(r)$, we obtain for ψ and χ

$$\psi'' + \lambda\varphi = 0, \qquad r^2\chi'' - r(\theta'^2 - 1)\chi' + \lambda(\theta'^2 - 1)\chi = 0 \tag{0.9}$$

where λ is an arbitrary constant. Initial conditions for (0.9) must be chosen in accordance with (0.7) and (0.8).

Notice that Equation (0.3) may be derived from the equations for cylindrical "automodel" motion if the process as a whole is adiabatic (see [4]).

1. The equation of the piston which supports the shock wave will be chosen in the form

$$\alpha_2 = \sigma(\alpha_1) \tag{1.1}$$

Solving the Hugoniot conditions in front of the shock wave, given by Equation (0.6), we obtain

$$a = |u_{1n}| = \frac{2(D^2 - \gamma)}{(\gamma+1)D}, \qquad p = \frac{2}{\gamma+1}D^2 - \frac{\gamma-1}{\gamma+1}, \qquad \rho = \frac{(\gamma+1)D^2}{(\gamma-1)D^2 + 2\gamma} \tag{1.2}$$

where $|u_{1n}|$ is the velocity modulus on the front of the wave. For the background in front of the wave, we assume $p = \rho = 1$, $u_1 = u_2 = 0$. Choosing the plus sign in the Equation (0.5), we obtain the following initial conditions for function θ

$$\theta = \frac{2}{\gamma-1}\sqrt{\gamma\frac{p}{\rho}}, \qquad \theta' = \frac{2(D-a)}{(\gamma-1)\theta(a)} \quad \text{for} \quad r = a \tag{1.3}$$

In this manner function $\theta(r)$ is uniquely determined from Equation (0.3), if D and γ are given. We note that Equation (0.2) is elliptic in the neighborhood of the wave front. To be elliptic it must satisfy the condition $1 - \theta'^2 > 0$. From (1.2) and (1.3) we may obtain

$$\theta'^2 = \frac{(\gamma-1)D^2 + 2\gamma}{\gamma(2D^2 - \gamma + 1)} \tag{1.4}$$

and $\theta'^2 < 1$ provided $D^2 > \gamma$, which is always valid because $\sqrt{(\gamma)}$ is the velocity of sound in a stagnant gas.

If the line $\alpha_2 = \sigma(\alpha_1)$ is to represent the motion of a piston, it is necessary and sufficient that the condition

$$D_\Pi = u_\Pi \tag{1.5}$$

be satisfied, where D_Π is the normal velocity of the piston and u_Π is the projection of the velocity vector on the normal to the piston. Using (0.6) and (1.1), condition (1.5) may be written in the form

$$\frac{\partial\alpha_2}{\partial t}-\sigma'\frac{\partial\alpha_1}{\partial t}=-u_1\left(\frac{\partial\alpha_2}{\partial x_1}-\sigma'\frac{\partial\alpha_1}{\partial x_1}\right)-u_2\left(\frac{\partial\alpha_2}{\partial x_2}-\sigma'\frac{\partial\alpha_1}{\partial x_2}\right) \tag{1.6}$$

Applying the method which was used in [1, 2] to the analysis of the basic system, it is easily shown that Equation (1.6) is equivalent to two equations:

$$u_2-\Delta_2+\sigma'(\Delta_1-u_1)=0 \tag{1.7}$$

$$p_{21}(u_1-\Delta_1)-\sigma' p_{12}(u_2-\Delta_2)-p_{11}(u_2-\Delta_2)+\sigma' p_{22}(u_1-\Delta_1)=0, \qquad p_{ij}=\frac{\partial\Delta_i}{\partial\alpha_j} \tag{1.8}$$

Further, using (0.1) we find that both Equations (1.7) and (1.8) are satisfied, if $u_{\Pi}\neq 0$, provided

$$\theta'(r)=0 \tag{1.9}$$

Hence it follows that Equation (0.3) together with the initial conditions (1.3) must be integrated in the direction of increasing r up to the value $r = d$, such that θ' becomes zero.

Let us note in passing that we may investigate pistons without slip. In that case, in addition to condition (1.9), the following requirement must be imposed upon Φ°

$$r\frac{\partial\Phi^{\circ}}{\partial r}-\Phi^{\circ}=0 \quad \text{for } r=d \tag{1.10}$$

This requirement together with the conditions in front of the shock wave leads to the following boundary value problem for the determination of the function $\chi_{\lambda}(r)$

$$a\chi_{\lambda}'(a)-\chi_{\lambda}(a)=0, \qquad d\chi_{\lambda}'(d)-\chi_{\lambda}(d)=0 \tag{1.11}$$

In the considerations to follow we restrict ourselves to the investigation of pistons with slip.

Consider the case when the shock front is symmetrical with respect to both axes of a closed smooth figure. Since it is closed and symmetrical, it is necessary to assume $\lambda = (2m)^2$, $m = 0, 1, 2, \ldots$ or $\lambda = 1$ in Equations (0.9) (we shall assume the lines $x_1 = 0$, $x_2 = 0$ to be rough walls).

Furthermore we consider a class of problems for which the velocity potential in the plane of the hodograph has the form

$$\Phi^{\circ}=c_1\chi_0(r)+c_2 r\psi_1(\varphi)+c_3\chi_2(r)\cos 2\varphi \tag{1.12}$$

This simplest example will serve to clarify the peculiarities that may arise in these types of flow with pistons and shock waves.

The term of the form $c_2 r\psi_1(\varphi)$ corresponds to a simple transformation of the origin of the coordinate system in the plane x_1, x_2 and it may be eliminated. From Equations (0.3) and (0.9) we have

$$r + \frac{\gamma - 1}{2}\theta\theta' = c\chi_0'(r) \qquad (c = \text{const}) \tag{1.13}$$

This condition means, however, that in Φ^0 the term of the form $c_1\chi_0(r)$ corresponds to the motion in time. Therefore, it may also be eliminated. In this way, Φ^0 may be chosen in the form

$$\Phi^0 = b\chi_2(r)\cos 2\varphi \tag{1.14}$$

The form of the shock front will be chosen at $t = 1$. If we assume $b = 0$ and $\Phi^0 \equiv 0$ we have the known solution of Sedov [4] for the expansion of a cylindrical piston at constant velocity from a point.

The function $\chi_2(r)$ may be found, upon determining $\theta(r)$ $(\lambda = 4)$, by numerical integration of Equation (0.9) with the initial condition

$$\chi_2(a) = a, \qquad \chi_2'(a) = 1 \tag{1.15}$$

which arises from condition (0.9). Upon determination of $\chi_2(r)$ and $\theta(r)$ the distribution of quantities in the x_1, x_2, t space is given by the following equations obtained from (0.4)

$$\begin{aligned} x_1 &= \left(r + \frac{\gamma-1}{2}\theta\theta'\right)t\cos\varphi + b\left(\cos\varphi\cos 2\varphi\chi_2'(r) + 2\sin\varphi\sin 2\varphi\frac{\chi_2(r)}{r}\right) \\ x_2 &= \left(r + \frac{\gamma-1}{2}\theta\theta'\right)t\sin\varphi + b\left(\sin\varphi\cos 2\varphi\,\chi_2'(r) - 2\cos\varphi\sin 2\varphi\frac{\chi_2(r)}{r}\right) \end{aligned} \tag{1.16}$$

where $r = a$ corresponds to the shock front and $r = d$ to the piston. Let

$$J(r, \varphi, t) = \begin{vmatrix} \partial x_1/\partial r & \partial x_1/\partial\varphi \\ \partial x_2/\partial r & \partial x_2/\partial\varphi \end{vmatrix} \tag{1.17}$$

Using (1.16), we represent $j(r, \varphi, t)$ in the form

$$J(r,\varphi,t) = \frac{\theta^{12} - 1}{r}\left[t\left(r + \frac{\gamma-1}{2}\theta\theta'\right) - b\cos 2\varphi\left(-\chi_2' + 4\frac{\chi_2}{r}\right)\right]^2 - b^2\frac{4}{r}\sin^2 2\varphi\left(\chi_2' - \frac{\chi_2}{r}\right)^2 \tag{1.18}$$

From (1.18) it follows that for sufficiently small but finite values of b for all r and φ and $t \geqslant 1$, the Jacobian $J(r, \varphi, t)$ does not vanish. By the same token the solution in the region between the shock front and the piston, valid for $t \geqslant 1$, is fully determined.

It should be noted that for $t < 1$ a solution $J(r, \varphi, t) \neq 0$ cannot be guaranteed. Whereas in the solution of Sedov the piston begins to move from a point and at the initial instant the piston and the shock front are coincident, it is not permissible to retrace the course of the wave and the piston "back" to the coincidence since for a certain t, $J(r, \varphi, t)$ vanishes. This situation is natural, since in the case of expansion of a cylindrical piston from a given radius other than zero a motion arises at the initial instant which in general is not self similar; only for sufficiently large t does it acquire similarity. In the present case, the flow arising immediately after expansion of some curvilinear cylindrical surface which is also a constant pressure surface, likewise in general does not belong to the types of flow with linear characteristics under consideration here. It is expected that only some time after the start of the motion will it evolve into the type treated here. It will be noted that the solution obtained for $t \to \infty$ transforms into the solution of Sedov.

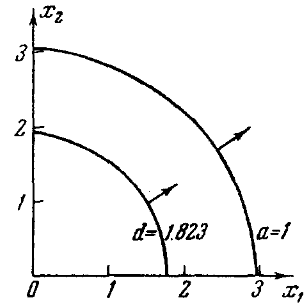

Fig. 1.

We shall give a specific example of the solution. Assume $D = 3$, $\gamma = 3$, $b = 0.03$. The quantity b was chosen small so that in the region between the piston and the shock wave the Jacobian $J(r, \varphi, t)$ for $t > 1$ will not vanish. The form of the wave and the piston in this case does not differ appreciably from the circular form. At a particular instant namely for $t = 1$ the points (2.97, 0), (0, 3.025) belong to the shock wave and the points (1.75, 0), (0, 1.895) to the piston respectively. Figure 1 shows the graph of the location of the front of the shock wave ($a = 1$) and of the piston ($d = 1.823$) for $t = 1$.

2. In contrast with the above considerations we assume now that ahead of the front of the detonating wave we have $p = 0$. From the Chapman-Jouguet condition

$$u_{1_n} + c = D \tag{2.1}$$

(the detonating wave is assumed to be normal) and the Hugoniot conditions we obtain along the detonating wave $\alpha_2 = f(\alpha_1)$ the following equations

$$u_{1n} = \frac{D}{\gamma + 1}, \qquad c = \frac{\gamma D}{\gamma + 1} \qquad \text{for } \alpha_2 = f(\alpha_1) \tag{2.2}$$

In Equations (2.1) and (2.2) u_{1_n} denotes the wave of the component of the velocity vector normal to the wave.

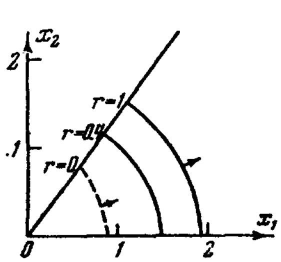

Fig. 2.

Two possibilities arise from Equation (0.5)

$$(1)\ D = -\frac{D}{\gamma + 1} - \gamma k, \qquad k = -\frac{\gamma + 1}{\gamma(\gamma + 2)} D \tag{2.3}$$

$$(2)\ D = \quad \frac{D}{\gamma+1} + \gamma k, \qquad k = \frac{D}{\gamma+1} \tag{2.4}$$

$$(k = r\theta' = \mathrm{const}$$

Behind the front of a normal propagating detonating wave, i.e. for the one-dimensional case the velocity drops to zero, while the pressure and the velocity of sound reach constant values. It is necessary, therefore, to consider the case of (2.4), because the sign of k characterizes an increase or decrease of the velocity of sound behind the front of the wave.

In this case we have

$$1 - \theta'^2 = 1 - \frac{k^2}{u_{1\ n}^{2}} = 0 \tag{2.5}$$

This means that the front of the detonating wave satisfies Equation (0.2) for the parabolic case. In this way we have the following Cauchy problem for Equation (0.3)

$$\theta = \frac{2\gamma D}{\gamma^2 - 1}, \quad \theta' = 1 \quad \text{for } r = \frac{D}{\gamma+1} \tag{2.6}$$

It can be shown further that in the neighborhood of the front on the side of decreasing r the Equation (0.2) is of the hyperbolic type, i.e. $(1 - \theta'^2) < 0$. Indeed, we obtain from (0.3)

$$\theta'' = -\frac{(\gamma+1)^2}{2\gamma D} < 0 \quad \text{for } r = u_{1n} \tag{2.7}$$

By numerical integration of (0.3) for various γ and D it can be shown that Equation (0.2) remains hyperbolic up to the line $r = 0$, which in the one-dimensional case is the line of a weak discontinuity.

We shall investigate the asymptotic representation of the function $1 - \theta'^2$ in the neighborhood of the line $r = 0$ (see, for example, [7]). Assuming that $\theta' \neq 0$ for $r \sim 0$, we have from 0.3)

$$r\theta'' - \theta' - \theta'^2 \sim 0 \tag{2.8}$$

we obtain by integration

$$\theta'^2 - 1 \sim B\sqrt{r} \qquad (B = \mathrm{const} > 0) \tag{2.9}$$

In this way the first derivative θ' is seen to be continuous at the transition through the line $r = 0$ and all higher derivatives become infinite. On the line $r = 0$, we have $\theta'^2 - 1 = 0$, hence this line also corresponds to the parabolic case of equation (0.2). In certain cases it may indicate a weak discontinuity, behind which the gas is stagnant and has constant density and pressure, analogous to the one-dimensional problem investigated by Zel'dovich. The one-dimensional solution for the cylindrical case is obtained if we assume $\Phi^{\circ} \equiv 0$.

The potential Φ°, in general, is determined uniquely by conditions (0.7) and (0.8) but additional conditions are needed in order to select a desired solution. We shall show that for $\lambda \neq 0$ and $\lambda \neq 1$ the solutions of Equation (0.9) have a singularity at $r = 0$ and that $\chi'_{\lambda}(r)$ are not bounded, so that in order to obtain solutions with a weak discontinuity, moving behind the detonating wave, it is necessary in general to construct a boundary value problem for the function χ_{λ}. The function $\chi_{\lambda}'(r)$ then must be bounded at zero.

Using (2.9) we obtain from (0.9) for χ_{λ} at $r \sim 0$ equation

$$r^{3/2}\chi''_{\lambda} - Br\chi'_{\lambda} + \lambda B\chi_{\lambda} = 0 \tag{2.10}$$

Let us introduce function M defined by the formula $\chi_{\lambda} = r^{\lambda}M$. From (2.10) we obtain for M equation

$$r^2 M'' + (2\lambda r - Br^{3/2}) M' + \lambda(\lambda - 1) M = 0 \tag{2.11}$$

At $r \sim 0$ we may consider instead of (2.11) the equation

$$r^2 M'' + 2\lambda r M' + \lambda(\lambda - 1) M = 0 \tag{2.12}$$

Equation (2.12) is Euler's equation. Looking for a solution in the form $M - r^{m}$, we obtain two possibilities for m: $m_1 = -\lambda$ and $m_2 = -\lambda + 1$.

Thus, the two linearly independent solutions of Equation (2.10) near zero have the form

$$\begin{aligned} \chi^{(1)} &= A^{(1)}r + 0(r), \qquad A^{(1)} = \text{const} \neq 0 \\ \chi^{(2)} &= A^{(2)} + 0(1), \qquad A^{(2)} = \text{const} \neq 0 \end{aligned} \tag{2.13}$$

But if $\chi_{\lambda}(0) \neq 0$ then it follows from (2.10) that χ_{λ}' is not bounded at zero.

Thus, in order to obtain solutions valid in the region between the detonating wave and the weak discontinuity, which separates the region of disturbed motion from the stagnant gas, we must specify for the functions χ_{λ}, which appear in the expression of the velocity potential in the hodograph plane

$$\Phi^{\circ} = \sum_{\lambda} a_{\lambda}\psi_{\lambda}(\varphi)\chi_{\lambda}(r) \qquad (a_{\lambda} = \text{const}) \tag{2.14}$$

the following boundary conditions

$$\chi_{\lambda}(0) = 0, \qquad a\chi'_{\lambda}(a) - \chi_{\lambda}(a) = 0 \tag{2.15}$$

When reducing Equation (0.9) for χ_λ to the self-adjoint form, we obtain

$$(p(r)\chi'_\lambda)' + \lambda q(r)\chi_\lambda = 0 \tag{2.16}$$

where

$$p(r) = \exp\left(-\int_a^r \frac{\theta'^2 - 1}{r}\,dr\right) > 0, \qquad q(r) = \frac{\theta'^2 - 1}{r}\exp\left(-\int_a^r \frac{\theta'^2 - 1}{r}\,dr\right) \geqslant 0 \tag{2.17}$$

The function $p(r)$ is continuous at $[0, a]$ and $q(r)$ has a singularity only for $r = 0$ and $q(a) = 0$. Applying the methods of Chapter 6 of [6] for equations with singular points, we can derive the usual properties of systems of eigen values and eigen functions of the boundary value problem (2.15).

Further, in order to construct a particular solution for Φ° we take, in analogy to Section 1

$$\Phi^\circ = b\chi_\lambda(r)\cos\sqrt{\lambda}\varphi \tag{2.18}$$

where $\lambda \neq 1$ is the first eigen value of the boundary value problem (2.15) for the Equation (0.9). With $\theta(r)$ and $\chi_\lambda(r)$ known, the flow in the x_1x_2t space may be found from the equations

$$\begin{aligned} x_1 &= \left(r + \frac{\gamma - 1}{2}\theta\theta'\right)t\cos\varphi + b\left(\cos\varphi\cos\sqrt{\lambda}\varphi\chi'_\lambda(r) + \sqrt{\lambda}\sin\varphi\sin\sqrt{\lambda}\varphi\frac{\chi_\lambda(r)}{r}\right) \\ x_2 &= \left(r + \frac{\gamma - 1}{2}\theta\theta'\right)t\sin\varphi + b\left(\sin\varphi\cos\sqrt{\lambda}\varphi\chi'_\lambda(r) - \sqrt{\lambda}\cos\varphi\sin\sqrt{\lambda}\varphi\frac{\chi_\lambda(r)}{r}\right) \end{aligned} \tag{2.19}$$

Equations (2.19) are the equations of motion of the front of the detonation for $r = a$, $r + \varpi'(\gamma - 1)/2 = D$; the value $r = 0$ corresponds to the line of a weak discontinuity. Let us fix the initial location of the front of detonation at $t = 1$. In this case, we cannot assert that the lines of the weak discontinuity and the detonating wave coincided at some instant $t < 1$. Similarly to the statements in Section 1 it is expected that, after the initiation of the detonating wave along some curvilinear cylindrical surface, the flow at the initial instant does not belong to the class of motions with linear components here considered. After some time, however, it will evolve into that type.

If instead of stating the boundary problem for the function χ_λ, we required that χ_λ' by bounded at zero, then we might by solving Equation (0.9) for the Cauchy problem $\chi_\lambda'(a) = 1$, $\chi_\lambda(a) = a$ of λ for arbitrary χ_λ, obtain various flows behind the detonating wave. Their validity extends only to some limit line along which the hodograph of velocities degenerates. These solutions evidently may be used after numerical integration of some problems, in which case they are matched along the singular line with the constructed solutions.

Let us investigate further the behavior of $j_\lambda(r, \varphi, t)$ in the region between the detonating wave and the line of weak discontinuity. From (1.18) we have for arbitrary

$$J_\lambda(a, \varphi, t) = 0 \tag{2.20}$$

Let us show that behind the front of the wave $J_\lambda > 0$. Indeed for $r = a$ we obtain

$$\frac{\partial J_\lambda}{\partial r} = -\frac{1}{D}\frac{(\gamma+1)^2}{a\gamma}(Dt - b(\lambda-1)\cos\sqrt{\lambda}\varphi)^2 < 0 \tag{2.21}$$

Using the following relationships for $r \sim 0$

$$\chi''_\lambda \sim Br^{-1/2}, \qquad \chi'_\lambda - \frac{\chi_\lambda}{r} \sim Nr^{+1/2}, \quad -\chi'_\lambda + \lambda\frac{\varkappa_\lambda}{r} \sim K \quad (B, N, K = \text{const}) \tag{2.22}$$

for sufficiently small (but finite) B it may be shown that $J_\lambda(r, \varphi, t) > 0$ for $r \in [0, a]$, $t \geqslant 1$ and arbitrary φ. It may be shown further that the obtained solutions may be considered also in the interval of time $0 < t_0 < t < 1$, where at the instant t_0 an intersection of the normals to the line of the weak discontinuity takes place and the discontinuity arises.

Note also that there follows from (2.19) for $r = 0$ that the line of the weak discontinuity moves with constant normal velocity, equal to $1/2(\gamma - 1)\theta(0)$, namely with the local sound velocity. In conclusion, let us give a numerical example.

Assume that $p = A\rho^3$ which is the most frequently used equation of state in the theory of detonation. Let $D = 4$ and $p = 0$ in front of the detonation.

Determination of the first eigen number by the method of Galerkin, yields $\lambda = 10.66$. By numerical integration of (0.3) we find the function $\theta(r)$ with the initial conditions

$$\theta = 3, \quad \theta' = 1 \quad \text{for } r = 1 \tag{2.23}$$

In this case, as shown by the calculation, $\theta' > 1$ in the interval (0.1).

The velocity potential Φ° is chosen in the form

$$\Phi^\circ = 0.01\ \chi_\lambda(r)\cos 3.26\varphi \tag{2.24}$$

(for $b = 0.01$ the condition $j_\lambda(r, \varphi, 2.14) > 0$ is satisfied for all φ and $r \in (0.1)$). The lines

$$x_2 = 0, \qquad x_2 = \tan \frac{\pi}{2 \cdot 3.26} x_1$$

are considered to be rough walls. Figure 2 shows the location of the detonation wave ($r = 1$) the line of the weak discontinuity ($\dot{r} = 0$) and the line $r = 0.4$ at the instant $t = 2.14$.

As in Section 1, the form of the detonation wave and of the weak discontinuity in the constructed example differs insignificantly from the circular form. The distances of the end points of the detonation wave and the weak discontinuity to the origin of the coordinate system are equal to 1.895 (on the line $x_2 = 0$) and 1.845 for the detonating wave and 0.846 (on the line $x_2 = 0$) and 0.905 for the points on the weak discontinuity respectively.

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